

Achieving pole-law inflation: the extreme inflation

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Abstract

The pre-big bang's inflationary mechanism, when allowance is made for the rapid change of Newton's constant, is not actually of pole-law form . We give examples where pole-law inflation, which requires violation of the weak-energy condition, is possible but unlikely due to its very unstable character.

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Introduction

The pre-big bang scenario, inspired by superstring theory, is claimed to be an alternative inflationary universe model to that of the usual scalar potential driven one - for reviews see [1]. The expansion is said to now start at time $t \rightarrow -\infty$ and is of the form $a \sim (-t)^p$ with $p < 0$, often dubbed “super-inflation” in a string theory context where $p = -1/\sqrt{3}$ [1]. Later a branch change is expected to switch to the post-big bang, now a less expansive $a \sim t^{1/\sqrt{3}}$ solution, which could easily join to a conventional FRW expansion. But this is also just an example of kinetic or pole-law inflation which is expected to be a more general phenomena that occurs in various alternative gravity theories, and was earlier obtained with an induced gravity model [2], which in turn can be related to compactification of higher dimensional gravity theories [3].

Actually these claims of pole-law inflation occurring in alternative gravity models, and similarly in the pre-big bang case, are erroneous as I have explained in a previous paper [4]. There I suggested the pre-big bang solution was not actually inflationary, but rather was a contraction with respect to the Planck length: all the usual pole-law schemes had ignored this problem about the growing Planck length. Here I wish to explain what actually would need to be done to obtain pole-law inflationary behaviour and give some possible matter sources: some of which have already cropped up in certain string theories. By this means I hope to strengthen and clarify my criticism of the pre-big bang scenario which, with dilaton alone, is not an example of this pole-law type.

Failure of dilaton driven pre-big bang inflation

Consider a flat FRW universe model with a perfect fluid equation of state $p = (\gamma - 1)\rho$, the expansion is $a \sim |t|^{2/3\gamma}$. Now notice the string theory value $a \sim |t|^{-1/\sqrt{3}}$ is equivalent to a value of $\gamma = -2\sqrt{3}/3 \simeq -1$. Unlike conventional inflation which violates the strong-energy condition $0 \leq \gamma < 2/3$, we now have the stricter requirement of violating the weak-energy condition $\gamma < 0$, see eg.[5]. So the string theory violates the weak-energy condition and so it explains why pole-law inflation is present? Well no, we should be careful since in string theory the dilaton causes Newton’s constant G to run while the energy conditions are formulated in general relativity where the Newton’s constant is really constant. But we can transform the string theory effective action to one where Newton’s constant is fixed in the

so-called Einstein frame.

Consider the simple model that comes from the low-energy string theory with action [6]

$$S = \int d^4x \sqrt{-g} \exp(-\phi) \left(R - \omega(\partial_\mu \phi)^2 \right) . \quad (1)$$

We have only included the dilaton ϕ term as this is the fundamental component that is suggested can possibly drive an expansion. Including a cut off for the finite string size, as done by ref.[7], would not be relevant as to whether inflation occurs to give a large universe.¹ But of course ultimately one should work with the full string theory when properly formulated in 10, 11 dimensions. We note in passing that the dilaton's role appears weakened in 11 dimensional supergravity which might allow a more conventional inflation cf.[8]. An inflation driving field can now remain uncoupled to the dilaton and so keep its 'shallowness', so allowing a violation of the strong-energy condition.

In the mean time we stick with investigating the dilaton, which is thought a fairly general aspect of strings when starting from 10 dimensions.

Using a field redefinition $\Phi = \exp(-\phi)$ the action can be rewritten in the more usual Brans-Dicke form

$$S = \int d^4x \sqrt{-g} \left(\Phi R - \frac{\omega}{\Phi} (\partial_\mu \Phi)^2 \right) . \quad (2)$$

Duality symmetry of string theory requires $\omega = -1$ [9] but we keep this ω term general for now.

By means of a conformal transformation to new quantities denoted by tildes, such that the new metric becomes-see eg. [10,11],

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad (3)$$

and where $\Omega^2 = \Phi$, we can find an equivalent action to expression (2). This can be expressed as

$$S = \int d^4x \sqrt{-\tilde{g}} \left(R(\tilde{g}) - 1/2(\tilde{\nabla}\sigma)^2 \right) , \quad (4)$$

¹ One can alternatively formulate the question as to whether inflation can drive the scale factor massively bigger than the string length scale l_s given the value of a fixed Newton's constant (here l_s is related to the Planck length by the string coupling constant g i.e. $l_p = gl_s$ but which are equivalent for strong coupling $g \sim 0(1)$ when inflation should finish and a branch change occur).

where the scalar field σ is defined from [10,11]

$$\Phi = \exp(\beta\sigma) \quad , \quad (5)$$

and $\beta^2 = 1/(2\omega + 3)$. This action (5) is simply that of a massless scalar field whose field equations with a FRW metric are,

$$H^2 = \dot{\sigma}^2 \quad (6)$$

$$\ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} = 0 \quad \Rightarrow \quad \dot{\sigma}^2 = \frac{A}{a^6} \quad (A = \text{constant}) \quad (7)$$

Newton's constant is now fixed (to unity) and the matter field is just that of a stiff equation of state $\gamma = 2$. So we have now apparently lost the presence of $\gamma < 0$ that is expected to drive a pole-law inflation. So in the original string frame we have $\gamma \simeq -1$, while in the Einstein frame $\gamma = 2$, so we have inflation in the string frame only and that's enough surely? Well not really, recall that inflation requires gravity to become repulsive unlike its usually attractive behaviour. This I contend should be a fundamental requisite when claiming inflation is present. Further, this aspect of inflation should be conformally invariant. Now why do I say gravity is remaining attractive? Newton's constant G in the pre-big bang phase starts small and grows as the dilaton increases [1,4] such that $G = \exp(\phi)$ so always being positive and since there is no cosmological constant (or dilaton potential) present it has no possibility of being overwhelmed by any repulsive component. As gravity is attractive there is no possibility of naturally getting a large universe and by comparing the Planck length scale with the scale factor one finds indeed that the universe is collapsing with respect to this Planck length, or alternatively a failure to grow w.r.t. the string length scale [4]. Any ensuing branch change to the post-big bang phase would still require the fixing of arbitrary constants (like A above) to be large to 'force' the well known mismatch of scales in the usual big bang model: or else requiring a further period of inflation, so really negating the reason for the pre-big bang phase.

Use of collapsing universe phase

Although collapsing universes do not inflate they do provide the possibility of providing a suitable fluctuation spectrum for later galaxy formation in the following expansionary phase. By altering the matter content one can change the density fluctuation spectrum. As we have seen the pre-big bang is a stiff fluid which gives a "blue spectrum": larger fluctuations on smaller

scales [12]. By reducing γ one obtains more power on larger scales and by the time of a dust ($\gamma = 1$) matter source one is getting a more favoured scale invariant spectrum [13]. However the use of contraction phases just transforms the ‘fine tuning’ problems of the usual big bang to an equivalent earlier question of why the universe started large and with the further requirement that one needs a bounce to connect to an expansion phase cf.[4]. Likewise, contraction also explains the resolution of the horizon and flatness problems, but only by fiat, by starting initially with a large homogeneous universe cf.[14].

In summary, the dilaton driven phase is not inflationary in the sense of generating a large universe. Rather it is just contracting w.r.t. Planck or string scale, which still begs the question why did the universe start so large initially? Collapsing solutions do allow fluctuations to ‘leave the horizon’ and seed future galaxy formation in an ensuing expansionary phase. By altering the equation of state in the collapsing phase, now for matter satisfying the strong-energy condition, the spectrum can be altered to fit requirements.

True pole-law inflation

How do we obtain a real pole-law inflation and not just a contraction phase. Recall that for a scalar field ϕ the definition of γ is given by

$$\gamma = \frac{2\dot{\phi}^2}{\dot{\phi}^2 + V(\phi)} \quad (8)$$

so to obtain $\gamma < 0$ we require, since $\dot{\phi}^2 > 0$ that $V(\phi) < 0$, that is a negative potential with $|V(\phi)| > \dot{\phi}^2$. By using the ideas of ref.[15,16] that one can run Einstein’s equations in reverse by first fixing the required behaviour of the scale factor, here say $a \sim |t|^{-1}$, we can derive the required scalar potential. Here it would be of the form of a negative exponential $V(\phi) \sim -\exp(\phi)$. Such a simulation only applies for a certain time before the field falls away to $\phi \rightarrow \infty$ [16,17]. This behaviour can be contrasted with an open anti-DeSitter space where the scale factor goes like $a \sim \cos(t)$, where there is also some expansion for time going from $-\pi/2$ to zero, and indeed anti-DeSitter also violates the weak-energy condition.

Negative exponential potentials might occur in the low energy limit of 11 dimensional supergravity theories cf. [18]

Some other sources can violate the weak-energy condition, for example the Brans-Dicke model but now with $\omega < -3/2$. The original induced gravity

matter source [2] that was supposed to give pole-law inflation also only caused a contraction ² since only values of the parameter $\epsilon > 0$ that gave conformal equivalence to a massless scalar field chosen. Taking more extreme values of ϵ or $\omega < -3/2$ seems to allow weak-energy condition violation but are unlikely to give stable solutions [11]. These weak-energy violating values have also been considered for the support of transversable Lorentzian wormholes [19] which by careful engineering can be made stable, but this is hardly feasible for cosmological solutions.

Higher order gravity theories can give negative potential in their conformally related scalar field models provided one takes ‘wrong signs’ in front of the coefficients of the higher order Ricci scalar eg. R^2 terms [20,21].

Also Lagrangians of the form $\mathcal{L} = \ln(1 + R)$ or $\exp(\lambda R)$, give negative, often exponential, potentials [20]. Certain non-analytic functions of Ricci tensors terms might be also be applicable [22].

Higher order corrections to string theory give Gauss-Bonnet terms that were hoped could amend the end of any pre-big bang phase [23]. These terms can also violate the weak-energy condition but are known to be unstable [11,24], especially because of Ricci tensor squared terms which are susceptible to growing anisotropy divergences [25]: they further apparently cause runaway black hole production [23].

It would seem preferable to just work with higher order corrections with ‘correct signs’ that can still violate the strong-energy condition: such schemes have been presented in [26].

A non-minimally coupled scalar field with sufficiently large field causes a violation of the weak-energy condition in that Newton’s constant effectively changes sign. The so-called rebouncing behaviour of ref.[27] is closely related to the possibility of pole-law expansion.

Bulk viscosity could in theory gives an effective $\gamma < 0$, see eg.[28], although it is suggested that it might not even give a negative pressure i.e. $\gamma < 2/3$ [29].

Coupling together of scalar fields such as a dilaton and axion field [30] can also violate the weak-energy condition, there they obtained an effective equation of state $\gamma = -2$ and worked only with the rapid contraction phase which occurs for positive $t > 0$, so still starting with a usual, time equal to

² It is straightforward to show, using the same argument as ref. [4], that this contraction occurs for any $p < 1$ so including all the pole-law behaviour $p < 0$.

zero, big bang model.

Trying to work with weak-energy violating inflation is likely to be fraught with instabilities cf.[11] and possible divergences in anisotropies as in the non-minimally coupled scalar field [27]. To use a particular example one would need to consider the stability carefully and arrange a suitable mechanism to end such inflation: at present no example seems remotely realistic.

In Fig.(1) we follow how the expansion rate becomes faster as the strong-energy condition is violated in conventional inflationary models. As the weak-energy condition also becomes violated the behaviour ‘switches over’: expansion now at negative times and rapid contraction for positive times. Incidentally, this is an example of *fragility* when going to negative γ : small changes in equation of state giving large changes in behaviour [31,16]. The extreme expansion in the scale factor only happens for $|t| < 1$ so requiring going infinitesimally close to the still singular behaviour ($\dot{a} \rightarrow \infty$ as $t \rightarrow 0_-$) at time zero, here an arbitrary constant. One is needing to work within the quantum gravitational regime to drive the expansion: this requires a correct version of quantum gravity, here string theory, unlike ordinary inflation which can still be relied on to proceed for below Planck scale values.

If one wishes to use the expansion at negative times you still need to match to a regular expansion now at positive time. Getting from one domain to another seems an added complication as the time parameter has to be carefully matched during this transition. If the weak-energy condition becomes satisfied too soon the universe will start collapsing for negative times (or in general for time below an arbitrary constant). What sets the time parameter correctly is an added complication over regular potential driven inflation, although quantum gravitational effects might give some hope of its resolution. The growing Hubble parameter, also a feature of weak-energy violating inflation, is also a problem since there are strong limits on the allowed size of the Hubble parameter to avoid gravitational waves [32] and the growing Hubble parameter will still eventually result in a curvature singularity.

Although weak-energy violating inflation remains a method of last resort, with its extreme expansion and tendency for instabilities, we suspect it is likely to prove ‘too hot to handle’. Searching string theory for strong-energy violating inflationary mechanisms, either by preventing the dilaton from rolling, or from higher order gravity corrections with stable signs, seems a more realistic endeavor.

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Figure Captions

Figure 1).

Expansion rate for various matter sources

The scale factor a against time t is plotted for matter sources ranging from strong-energy condition being satisfied (I) to weak-energy condition being violated (IV). Plot (I) radiation ($\gamma = 4/3$) : (II) coasting solution ($\gamma = 2/3$), where strong-energy condition is just being violated : (III) exponential expansion ($\gamma = 0$), the most extreme inflation with only strong-energy condition violated : (IV) Pole-law expansion ($\gamma = -2/3$), now weak-energy condition is violated to give extreme expansion for negative times or rapid contraction for positive times. Note that use of pole-law expansion to generate a large scale factor requires going closer than Planck time to the singular behaviour at, here, $t = 0$. Before a switch to a more normal expansionary behaviour can intervene.

References

1. M. Gasperini and G. Veneziano, *Astropart. Phys.* 1 (1993) p.317.
M. Gasperini, “Birth of the Universe in String cosmology”, preprint gr-qc/9706037.
S.J. Rey, “Recent progress in string inflationary cosmology”, preprint hep-th/9609115
J.J. Levin, “Gravity driven inflation”, preprint gr-qc/9506017.
Available at pre-big bang web site:
<http://www.to.infn.it/teorici/gasperini/>
2. M.D. Pollock and D. Shahdev, *Phys. Lett. B* 222 (1989) p.12.
3. D. Sahdev, *Phys. Lett. B* 137 (1984) p.155.
Phys. Rev. D 30 (1984) p.2495.
4. D.H. Coule, *Class. Quant. Grav.* 15 (1998) p.2803.
5. S.W. Hawking and G.F.R. Ellis, “The large scale structure of Space-time”, (Cambridge: Cambridge University Press) 1973.
M. Visser, “Lorentzian Wormholes”, (New York: AIP Press) 1995.
6. E.S. Fradkin and A.A. Tseytlin, *Phys. Lett. B* 158 (1985) p.316.
C.G. Callan, D. Friedan, E.J. Martinec and M.J. Perry, *Nucl. Phys. B* 262 (1985) p.593.
C. Lovelace, *Nucl. Phys. B* 273 (1985) p.135.
7. N. Kaloper, A. Linde and R. Bousso, preprint hep-th/9801073.
8. N. Kaloper, I.I. Kogan and K.O. Olive, *Phys. Rev. D* 57 (1998) p.6242.
9. G. Veneziano, *Phys. Lett. B* 263 (1991) p. 287.
10. S. Kalara, N. Kaloper and K.A. Olive, *Nucl. Phys. B* 341 (1990) p.252.
11. G. Magnano and L.M. Sokolowski, *Phys. Rev. D* 50 (1994) p. 5039.
L.M. Sokolowski, preprint gr-qc/9511073.
12. J. Hwang and H. Noh, *Phys. Rev. D* 54 (1996) p. 1460

13. M. Gasperini and G. Veneziano, Mod. Phys. Lett. A 8 (1993) p.3701.
14. D. Wands, preprint gr-qc/9809062.
15. G.F.R. Ellis and M.D. Madsen, Class. Quant. Grav. 8 (1991) p.667.
16. G.F.R. Ellis, J.E.F. Skea and R.K. Tavakol, Europhys. Lett. 16 (1991) p.767.
17. G.F.R. Ellis, D.H. Lyth and M. Mijić, Phys. Lett. B 271 (1991) p.52.
18. M.S. Bremer, M.J. Duff, H. Lu, C.N. Pope and K.S. Stelle, preprint hep-th/9807051
S.W. Hawking and H.S. Reall, preprint hep-th/9807100.
19. K.K. Nandi and A. Islam, Phys. Rev. D 55 (1997) p.2497.
A.G. Agnese and M. La. Camera, Phys. Rev. D 51 (1995) p.2011.
20. J.D. Barrow and S. Cotsakis, Phys. Lett. B 214 (1988) p.515.
ibid, Phys. Lett. B 258 (1991) p.299.
21. K. Maeda, Phys. Rev. D 37 (1989) p.858.
22. E. Brüning, D.H. Coule and C. Xu, Gen. Rel. Grav. 26 (1994) p.1193.
23. S. Kawai, M. Sakagami and J. Soda, preprint gr-qc/9802033.
24. L.M. Sokolowski, Z. Golda, M. Litterio and L. Amendola, Int. J. Mod. Phys. A 6 (1991) p.4517.
25. A.L. Berkin, Phys. Rev. D 44 (1991) p.1020.
26. J. Ellis, N. Kaloper, K.A. Olive and J. Yokoyama, preprint hep-th/9807482.
27. T. Morishima and T. Futamase, preprint gr-qc/9808040.
28. R. Maartens, Class. Quant. Grav. 12 (1995) p.1455.
W. Zimdahl, Phys. Rev. D 53 (1996) p.5483.
29. T. Pacher, J.A. Stein-Schabes, M.S. Turner, Phys. Rev. D 36 (1987) p.1603.

- 30. F.G. Alvarenga and J.C. Fabris, *Class. Quant. Grav.* 12 (1995) p. L69.
- 31. R.K. Tavakol and G.F.R. Ellis, *Phys. Lett. A* 130 (1987) p.217.
- 32. V.A. Rubakov, M.V. Sazhin and A.V. Veryaskin, *Phys Lett. B* 115 (1982) p.189.
L.F. Abbott and M.B. Wise, *Nucl. Phys. B* 244 (1984) p.541.